AP® CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 5

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

(a) $2yy' = y + xy'$
$(2y - x)y' = y$
$y' = \frac{y}{2y - x}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{solves for } y' \end{cases}$

(b) $\frac{y}{2y - x} = \frac{1}{2}$
$2y = 2y - x$
$x = 0$
$y = \pm\sqrt{2}$
$(0, \sqrt{2}), (0, -\sqrt{2})$

$2 : \begin{cases} 1 : \frac{y}{2y - x} = \frac{1}{2} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{y}{2y - x} = 0$
$y = 0$
The curve has no horizontal tangent since
$0^2 \neq 2 + x \cdot 0$ for any x.

$2 : \begin{cases} 1 : y = 0 \\ 1 : \text{explanation} \end{cases}$

(d) When $y = 3$, $3^2 = 2 + 3x$ so $x = \frac{7}{3}$.
$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{y}{2y - x} \cdot \frac{dx}{dt}$
At $t = 5$, $6 = \frac{3}{6 - \frac{7}{3}} \cdot \frac{dx}{dt} = \frac{9}{11} \cdot \frac{dx}{dt}$
$\left.\frac{dx}{dt}\right|_{t=5} = \frac{22}{3}$

$3 : \begin{cases} 1 : \text{solves for } x \\ 1 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$