AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$

$\left(4y^3 + 4\right)\dfrac{dy}{dx} = -2x - 2$

$\dfrac{dy}{dx} = \dfrac{-2(x+1)}{4\left(y^3+1\right)} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$

2 : { 1 : implicit differentiation; 1 : verification }

(b) $\left.\dfrac{dy}{dx}\right|_{(-2,\,1)} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$

Tangent line: $y = 1 + \dfrac{1}{4}(x+2)$

2 : { 1 : slope; 1 : tangent line equation }

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

3 : { 1 : $y = -1$; 1 : substitutes $y = -1$ into the equation of the curve; 1 : answer }

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

The curve crosses the x-axis where $y = 0$.

$(-1)^2 + 2(-1) + 0^4 + 4\cdot 0 \neq 5$

No, the curve cannot have a horizontal tangent where it crosses the x-axis.

2 : { 1 : works with $x = -1$ or $y = 0$; 1 : answer with reason }