

1. A particle moves along the  $y$ -axis with velocity given by  $v(t) = t \sin(t^2)$  for  $t \geq 0$ .
  - (a) In which direction (up or down) is the particle moving at time  $t = 1.5$ ? Why?
  - (b) Find the acceleration of the particle at time  $t = 1.5$ . Is the velocity of the particle increasing at  $t = 1.5$ ? Why or why not?
  - (c) Given that  $y(t)$  is the position of the particle at time  $t$  and that  $y(0) = 3$ , find  $y(2)$ .
  - (d) Find the total distance traveled by the particle from  $t = 0$  to  $t = 2$ .

<p>(a) <math>v(1.5) = 1.5 \sin(1.5^2) = 1.167</math> Up, because <math>v(1.5) &gt; 0</math></p> <p>(b) <math>a(t) = v'(t) = \sin t^2 + 2t^2 \cos t^2</math> <math>a(1.5) = v'(1.5) = -2.048</math> or <math>-2.049</math> No; <math>v</math> is decreasing at 1.5 because <math>v'(1.5) &lt; 0</math></p> <p>(c) <math>y(t) = \int v(t) dt</math> <math>= \int t \sin t^2 dt = -\frac{\cos t^2}{2} + C</math> <math>y(0) = 3 = -\frac{1}{2} + C \implies C = \frac{7}{2}</math> <math>y(t) = -\frac{1}{2} \cos t^2 + \frac{7}{2}</math> <math>y(2) = -\frac{1}{2} \cos 4 + \frac{7}{2} = 3.826</math> or <math>3.827</math></p> <p>(d) distance <math>= \int_0^2  v(t)  dt = 1.173</math> or <math>v(t) = t \sin t^2 = 0</math> <math>t = 0</math> or <math>t = \sqrt{\pi} \approx 1.772</math> <math>y(0) = 3</math>; <math>y(\sqrt{\pi}) = 4</math>; <math>y(2) = 3.826</math> or <math>3.827</math> <math>[y(\sqrt{\pi}) - y(0)] + [y(2) - y(\sqrt{\pi})]</math> <math>= 1.173</math> or <math>1.174</math></p>	<p>1: answer and reason</p> <p>2 <math>\left\{ \begin{array}{l} 1: a(1.5) \\ 1: \text{conclusion and reason} \end{array} \right.</math></p> <p>3 <math>\left\{ \begin{array}{l} 1: y(t) = \int v(t) dt \\ 1: y(t) = -\frac{1}{2} \cos t^2 + C \\ 1: y(2) \end{array} \right.</math></p> <p>3 <math>\left\{ \begin{array}{l} 1: \text{limits of 0 and 2 on an integral of } v(t) \text{ or }  v(t)  \\ \text{or} \\ \text{uses } y(0) \text{ and } y(2) \text{ to compute distance} \\ 1: \text{handles change of direction at student's turning point} \\ 1: \text{answer} \\ 0/1 \text{ if incorrect turning point} \end{array} \right.</math></p>
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